Ergodic chaos-based communication schemes

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Recent studies have shown the applicability of synchronized chaotic systems to the area of communications in different ways. At the same time synchronization based signal recovery and estimation of parameters severely suffer due to the presence of channel noise. By exploiting the ergodic properties of chaotic signals effectively, a simple technique called the mean-value method is introduced. This method is shown to be capable of estimating chaos system parameters from the transmitted chaotic signal efficiently for a low signal-to-noise ratio. A suitable demodulator has been designed for ergodic chaotic parameter modulation scheme for digital signal communication. Further, the mean-value technique incorporates a noncoherent receiver to recover analog information signal from the chaos masked signal efficiently. It is found that the proposed chaotic masking scheme is robust even in the presence of strong noise. In addition, this scheme has the potential advantage of a very simple hardware realization, which promises enhanced signal recovery performances.

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I. INTRODUCTION

The field of chaos-based communication has recently received a great deal of interest and a number of interesting techniques have been proposed. These techniques include chaos masking (CM) [1–6], chaos shift keying [7,8], chaotic spreading code [9,10], and chaotic parameter modulation [11–13]. Among these, chaotic masking and chaotic parameter modulation can be applied to modulate analog information signals and the rest of the schemes are mainly developed for digital communications. These techniques mainly utilize the inherent advantages by exploiting the nonperiodicity and unpredictable properties of chaotic signals to achieve spread-spectrum (SS) transmission of information signals. Although the concept of chaos-based communication is promising and there are many benefits of using a chaotic SS, so far very limited work has been carried out due to the main reason of poor performance of these schemes in the presence of channel or measurement noise.

Among the most promising chaotic communication schemes, the chaotic modulation or parameter modulation (CPM) approach [1,11–13] and the CM approach [1–6] are of great interest as they can be potentially applied to digital and analog spread-spectrum communications, respectively. The CPM stores the message signal in the bifurcation parameter of a chaotic system for SS communications. That is, by keeping the parameter in the chaotic regime, the output signal of the chaotic system is therefore wideband and noiselike, which can be used for SS transmission. Compared to conventional SS approaches based on spreading code, the CPM method does not require the complicated synchronization procedure for demodulation and has the potential of higher system capacity. However, the CPM communication system indeed needs a demodulator that can estimate the parameter of the transmitter chaotic system from the received chaotic signal accurately to decode the message signal. When measurement or channel noise exists, the conventional estimators are not efficient for designing a practical CPM communication system. Also, CM is a chaotic SS communication scheme that utilizes a noiselike chaotic signal from a self-synchronizing system to mask an analog information signal simply by adding the chaotic signal to the information signal [1–6,13]. By using an identical chaotic system with a suitable configuration at the receiver, the information signal can be extracted by subtracting the regenerated chaotic signal at the receiver from the original transmitted signal. Here the receiver system acts like a ''matched filter'' to filter out the message signal. Moreover, synchronization is possible only when the power level of the information signal is sufficiently smaller than that of the chaotic masking signal. Therefore, the inevitable additive noise from the channel may affect the synchronization and ensuing signal recovery. Thus the conventional CM communications based on chaos synchronization require very high signal-to-noise ratios (SNRs) and/or chaotic systems with self-synchronizing property [15–21]. Also, recent studies indicate that synchronization (coherent receiver) based signal recovery techniques usually suffer severely due to parameter mismatches between the transmitter and receiver systems. These results indicate that the conventional CM has limited advantages for communication applications [14–18].

However, by utilizing the good decorrelation properties of the chaotic signals, suitable noncoherent receivers are designed and tested to transmit digital information signals. Some of these designs are shown to be robust to channel noise [8,22,23]. But very limited work had been done in the development of suitable techniques to design viable noncoherent receivers for analog information signal recovery from the chaos masked signal [11]. Currently, most of the researches on the chaos masking modulation focus on the improvement of reconstruction or the recovery of the information signal with or without synchronization. Also, in addition to these approaches based on synchronizable chaotic receivers, several methods such as nonlinear forecasting systems [24], signal reconstruction [25,26], and adaptive filter approach [11] have been developed. The common feature of nonlinear forecasting systems and signal reconstruction methods is that a differential operator is a prerequisite of system design. But, the use of the differential operator is limited if the influence of channel noise is considered. Therefore, in order to make the CPM and CM schemes practical for a spread-spectrum digital and analog communication application, respectively, suitable new demodulation schemes have to be developed.

Recently, based on the ergodicity of the chaotic signal, a simple technique is introduced to estimate the chaotic parameters under a noisy environment [27]. Motivated by this technique, in this paper we propose a CM method for analog signal transmissions and a CPM method for digital communications. In Sec. II, the basic idea of the proposed ergodic theory for chaos-based communications and parameter estimation is discussed. In Sec. III, the performance of the analog spread-spectrum communication scheme based on the ergodic theory of chaos is considered. Section IV deals with the ergodic CPM based digital signal transmission scheme and its performance analysis. Finally, concluding remarks are discussed in Sec. V.

II. ERGODIC THEORY FOR CHAOS COMMUNICATIONS

Let f_θ be a chaotic map, Eq. (1), defined on some closed interval and θ be the control parameter. Let $\{x_t\}$ be a chaotic signal generated by f_θ as

$$x_t = f_\theta(X_{t-1}), \tag{1}$$

$$y_t = x_t = n_t,$$

where $X_{t-1} = [x_{t-1}, x_{t-2}, \ldots, x_{t-d}]^T$ is the d-dimensional state vector at time $t-1$ and n_t is a zero-mean additive white Gaussian noise (AWGN) process. For each $\theta \in [\theta_a, \theta_b]$, numerical experiment suggests that the map f_θ has an ergodic measure μ_θ. The chaotic signal $\{x_t\}$ is basically an orbit of f_θ with the initial condition x_0. According to the well-known Birkhoff ergodic theorem [28,29], the limit $\lim_{N\to\infty}(1/N)\Sigma_{i=1}^{N} x_t$ exists and is equal to the mean value $\int x \, d\mu_\theta(x)$. The limit is independent of the initial condition x_0 and depends only on the parameter θ. This function can be called the mean-value function $M(\theta)$ of the chaotic map f_θ. For example, a set of the ensemble estimations of the mean value of chaotic signals generated by two popular chaotic systems, namely, the Tent map defined by

$$T_\theta(x_{t-1}) = x_t = \theta - 1 - \theta|x_{t-1}|, \tag{2}$$

where $x \in [-1,1]$ and $\theta \in (1,2]$, and the Chebyshev map defined by

$$C_\theta(x_{t-1}) = x_t = \cos[\theta \cos^{-1}(x_{t-1})], \tag{3}$$

where $x \in [-1,1]$ and $\theta \in (1,2]$ are depicted in Figs. 1(a) and 1(b), respectively. The mean values are estimated based on 1000 trials with randomly selected initial conditions for the chaos signals generated from the Tent map and the Chebyshev map. The mean-value function of the Tent map is not monotonic over the entire parameter range but is numerically apparently monotonic increasing over the range $\theta \in [1.1,1.6]$. On the other hand, the mean-value function of

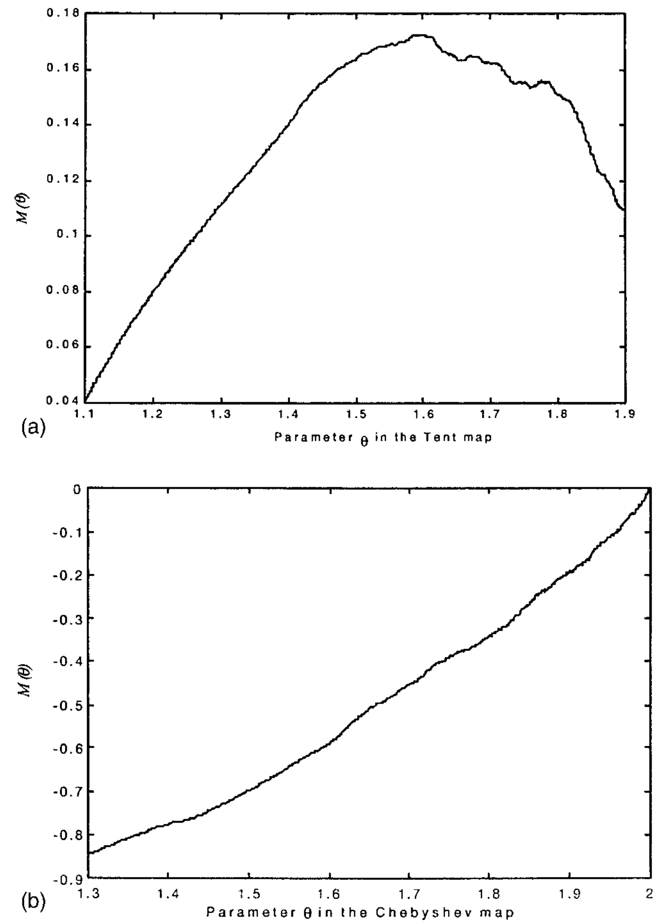

FIG. 1. (a) The mean-value function $M(\theta)$ of the Tent map. (b) The mean-value function $M(\theta)$ of the Chebyshev map.

the Chebyshev map apparently has a monotonic mean-value nature over the whole parameter range. Also, for many chaotic maps, we have an interesting observation of monotonic mean-value functions depending upon certain control parameter values. This implies that it is possible, at least in theory to estimate a particular parameter value θ_0 by the following procedure. Assume that $\{y_t\}$ is generated by Eq. (1) with $\theta = \theta_0$. First, we estimate the mean value $M_0 = M(\theta)$ from the received signal $\{y_t\}$. Second, we invert the function $M(\theta)$ to obtain an estimate of θ_0, i.e., $\hat{\theta}_0 = M^{-1}(M_0) = M^{-1}((M(\theta_0))$. Since $M(\theta)$ is numerically apparently monotonic, the existence of M^{-1} is guaranteed.

To avoid deriving the inverse mean-value function M^{-1} that may be difficult to obtain analytically, we can obtain $\hat{\theta}_0$ by solving the following optimization problem. Suppose that the mean-value function $M(\theta)$ is continuous and apparently monotonic on the interval $[\theta_a, \theta_b]$. If $M(\theta_0)$ is given, then θ_0 can be determined by finding the minimum of $D(\theta) = |M(\theta) - M(\theta_0)|$ for $\theta \in [\theta_a, \theta_b]$. Thus the ergodic theory based mean-value estimation algorithm can be summarized as follows:

(1) Compute an estimate of the mean value of the received signal $\{y_t\}$ using the ensemble average, that is, $\hat{M}(\theta_0) = (y_1 + y_2 + \cdots + y_N)/N = (1/N)\Sigma_{t=1}^{N} y_t$.

(2) Use the golden section search to locate the minimum

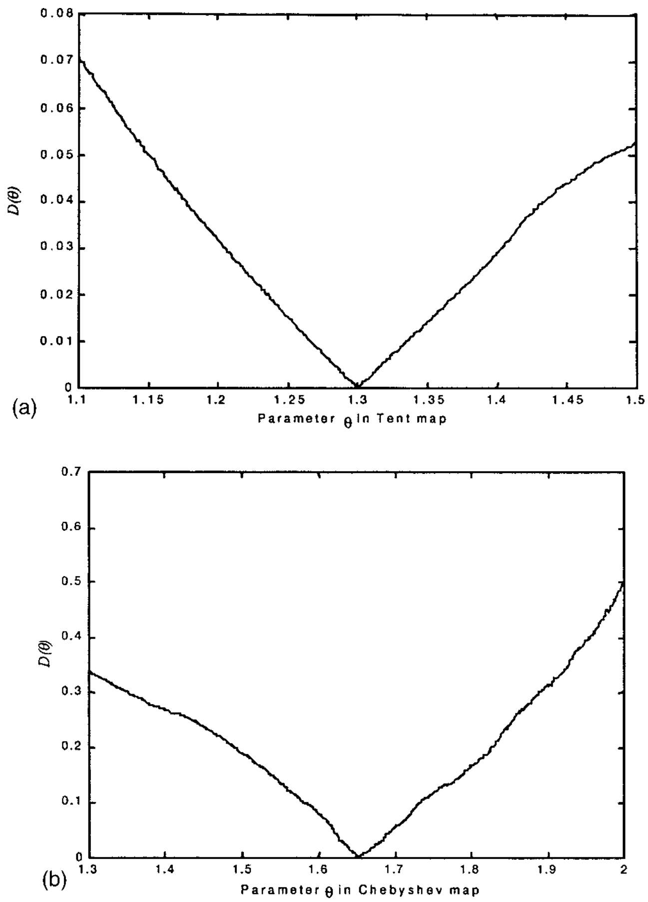

FIG. 2. The unimodal optimization function $D(\theta)$ for the Tent map with $\theta_0=1.3$. (b) The unimodal optimization function $D(\theta)$ for the Chebyshev map with $\theta_0=1.65$.

of $\hat{D}(\theta)=|\hat{M}(\theta)-\hat{M}(\theta_0)|$, where $\hat{M}(\theta)$ is a numerical approximation of $M(\theta)$ based on $\hat{M}(\theta_t)=1/N\Sigma_{t=1}^{N}x_t(\theta_i)$ and $\{x_t(\theta_i)|t=1,2,3,...\}$ is the data sequence generated by the dynamical system $x_t=f_\theta(X_{t-1})$ given in Eq. (1) with $\theta=\theta_i$.

In order to illustrate the effectiveness of the proposed method, Figs. 2(a) and 2(b) depict the numerically simulated objective function $D(\theta)$ for both Tent and Chebyshev maps, respectively. θ_0 in both the cases are selected randomly over the parameter range with an apparent monotonic mean-value function as shown in Figs. 1(a) and 1(b). As $D(\theta)$ in both Figs. 2(a) and 2(b) are unimodal functions, hence the global minima for both functions can be obtained easily. Since we do not have the analytical form of the $M(\theta)$ for these maps, we have to use the approximation $\hat{M}(\theta)$. To understand the accuracy of the approximation, we plot the estimation variance of the mean-value function of the Chebyshev map in a three-dimensional plot as a function of sample numbers as well as the additive noise variance as depicted in Fig. 3. It can be seen that the simulated performance is very close to the theoretical performance in which the asymptotic variance of the estimator $\hat{M}(\theta_0)$ is equal to $(\sigma_1^2+\sigma_2^2)/N$ and the asymptotic variance of the estimate $\hat{\theta}_0$ is equal to $[M^{-1}((M(\theta_0))]^2[(\sigma_1^2+\sigma_2^2)/N]$, where N is the number of

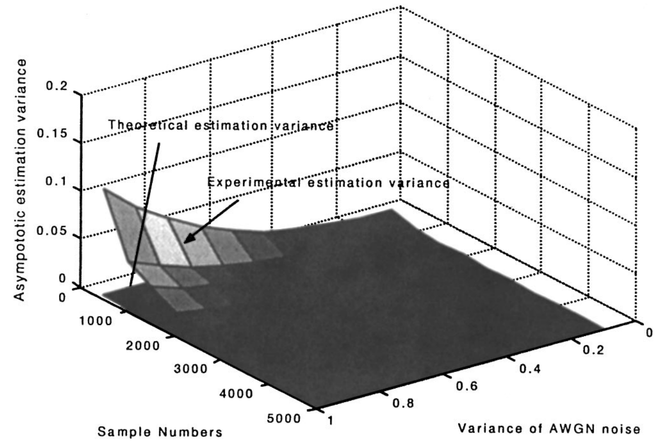

FIG. 3. Asymptotic estimation performance of the Chebyshev map versus the number of sample points and the variance of AWGN. The average power of the chaotic signal is assumed to be unity. Here computer simulation experiment performance versus theoretical estimation performance is presented.

sample points in the noisy chaotic signal, σ_1^2 is the chaotic signal power, and σ_2^2 is the noise power. Further, Monte Carlo simulation is carried out to evaluate the accuracy of the parameter estimates. The mean square error (MSE) in the parameter is evaluated versus various levels of the SNR. The range of SNR considered in our study is from -20 to 20 dB with an increment of 5 dB. Each MSE value is computed by using an average of 100 trials. First, we assume that we have a very long signal sequence, that is, N is very large for the signal set $\{y_t|t=1,2,...,N\}$. Since the proposed method relies on the ensemble average formula and as $N\rightarrow\infty$, the proposed method should show the ideal performance. We also implement two standard parameter estimation methods for comparison. The first one is the gradient search technique [30], which searches for an optimal estimate of θ along the direction of the gradient of the error function. The derived iterative equation is given by

$$\theta_t=\theta_{t-1}\sim\mu[y_t-f_{\theta_{t-1}}(y_{t-1})][df_\theta(y)/dy]|_{y=y_{t-1}},$$

where f_θ is the governing chaos map function C_θ. The second standard technique is the nonlinear least squares method based on the Gauss-Newton method, and our implementation of the second method is based on the MATLAB function NLINFIT. For the details of the Gauss-Newton method, the readers are referred to Ref. [31], which shows the iterative equations for the nonlinear least squares method. The MSE curves for the Chebyshev map are plotted in Fig. 4. Because of the high nonlinearity in the Chebyshev map, both standard methods are not quite effective even when the SNR is high. The proposed mean-value method apparently can improve the estimation accuracy significantly. In this experiment, when the signal sequence is very long, the estimated mean values of the signal and noise processes are almost equal to their exact values. The mean-value estimation method can estimate the parameter of the Chebyshev map accurately (around -55 dB) even when the SNR is -20 dB.

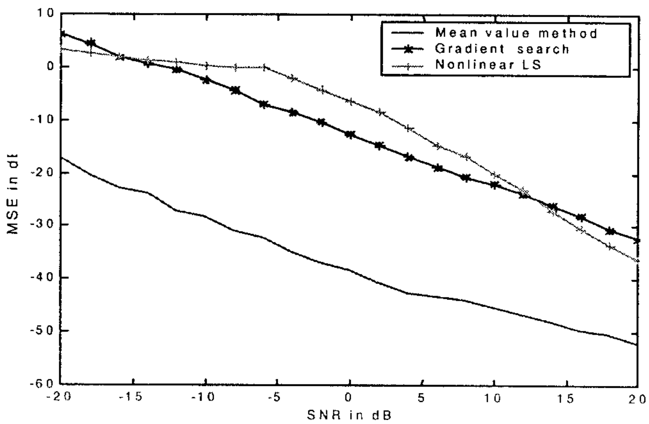

FIG. 4. Performance evaluation of various parameter estimation methods for the Chebyshev map under AWGN with nonexact zero mean. The parameter θ_0 is selected randomly on [1.3, 2].

III. ANALOG SPREAD-SPECTRUM COMMUNICATIONS USING ERGODIC CHAOS THEORY

In this section, we consider the applicability of ergodic theory of chaos based mean-value demodulation scheme based on the for the case of CM based analog spread-spectrum communication scheme. The basic block diagram of the CM communication system based on the mean-value estimation and demodulation is shown in Fig. 5. A mean-value estimator based on a windowed integrator is utilized to perform the mean-value estimation both at the transmitter and receiver. To enhance the performance of the noncoherent receiver, a normalized version of the chaotic signal is used as the masking signal. To do that, the chaotic carrier $z=x(n)=x_t$ is used from the Chebyshev map for $\theta=2.0$. The chaotic carrier z is normalized at the transmitter side by fixing its mean value, which approaches approximately zero within each integration time period T. That is,

$$x(t)=z-\frac{1}{T}\int_T z\,dt. \tag{4}$$

For actual transmission purposes the message signal $s(t)$ is added with the normalized chaotic signal $x(t)$ by considering the typical additive chaos masking scheme. From the transmitter, the signal $m(t)=x(t)+s(t)$ is transmitted through an AWGN channel to the noncoherent receiver. The signal $r(t)=m(t)+n(t)$ at the receiver is used for decoding the information signal and $n(t)$ is a zero-mean additive white Gaussian noise process. The demodulator processes the mean-value estimation within a windowed time period $T=T_s$ on the received noise-corrupted signal $r(t)$. The output of the mean-value demodulator is sampled with the sampling rate f_s, where $f_s=1/T_s$, and the sampled demodulated output is represented as

$$\begin{aligned} d(k) &= \frac{1}{T_s}\int_{kTs}^{(k+1)Ts} r(t)\,dt, \\ &= \frac{1}{T_s}\int_{kTs}^{(k+1)Ts} [s(t)+x(t)+n(t)]\,dt, \\ &= \frac{1}{T_s}\int_{kTs}^{(k+1)Ts} s(t)\,dt + \frac{1}{T_s}\int_{kTs}^{(k+1)Ts} x(t)\,dt \\ &\quad + \frac{1}{T_s}\int_{kTs}^{(k+1)Ts} n(t)\,dt, \\ &\approx \tilde{s}+\tilde{M}+\tilde{N}. \end{aligned} \tag{5}$$

Here, $\tilde{M}$ is the mean-value function of the normalized chaos masking signal, which is approximately equal to zero within each integration time period T_s and as the expectation of an AWGN process is zero, $\tilde{N}$ is also equal to zero. If the sampling rate $f_s=1/T_s$ satisfies the sampling theory, which is greater than twice the frequency of the information signal f, that is, $f_s \gg 2f$, then $\tilde{s}$ is the sampled value of the transmitted analog signal $s(t)$. Thus $d(k)$ corresponds to the sampled output $\tilde{s}$. After suitable filtering and amplification of the sampled output $\tilde{s}$, the recovered signal is denoted as e. The main advantages of the mean-value demodulator are as follows:

(i) The masking chaos signal is directly removed according to the knowledge of its first-order stochastic properties and no reconstruction of the chaos generation function at the receiver is needed.

(ii) During the demodulation process or the despreading process of the transmitted signal, the high power AWGN noise could be directly canceled due to its first-order stochastic property,

(iii) The demodulation procedure is very simple and practical, which is easy to be implemented experimentally.

The performance of the proposed CM communication system is discussed below. The chaos signal generated from the Chebyshev map is normalized first by removing its mean value within a certain time period T_s, and then the power or amplitude is adjusted to the required masking level. The normalization of the chaos signal at the transmitter side could reduce the recovery error at the receiver side and also reduce the complexity of the demodulator. The analog information signal is added directly with the normalized chaos signal at a certain power level. The masked signal is passed through a noisy channel to the receiver. At the receiver, the mean-value demodulator processes the noise-corrupted signal. In the demodulator, a simple windowed integrator is used as the mean-value estimator. The sampled output of the demodulator is seen as the recovered information signal. For the simu-

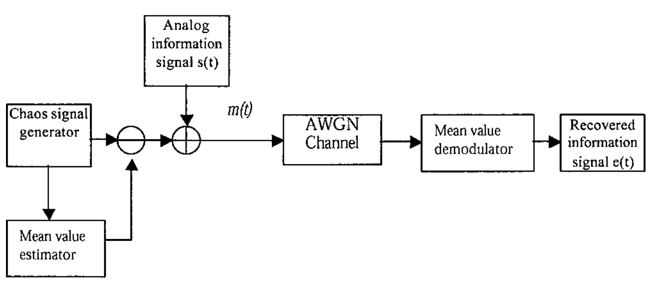

FIG. 5. Block diagram of the proposed chaos masking and mean-value demodulator based analog SS communication scheme.

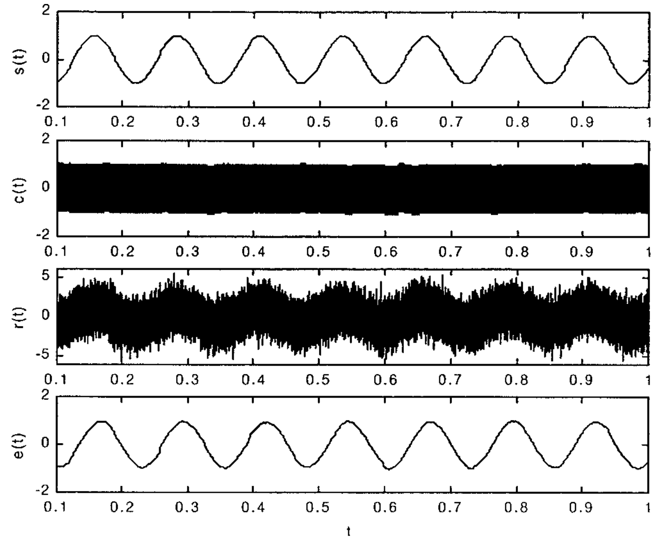

FIG. 6. Different kinds of signals observed in simulations when additive masking is used. Here $s(t)$ is the message signal $=\sin(2\pi ft)$, $f=50$ Hz, $c(t)$ is the normalized chaotic signal generated from the Chebyshev map, $r(t)=c(t)+s(t)+n(t)$ is the actual received noise-corrupted chaos masked signal, and $e(t)$ is the recovered signal for SNR$=0$ dB level. The time period of the windowed integrator $T_s=0.02$ s and $\theta=2$.

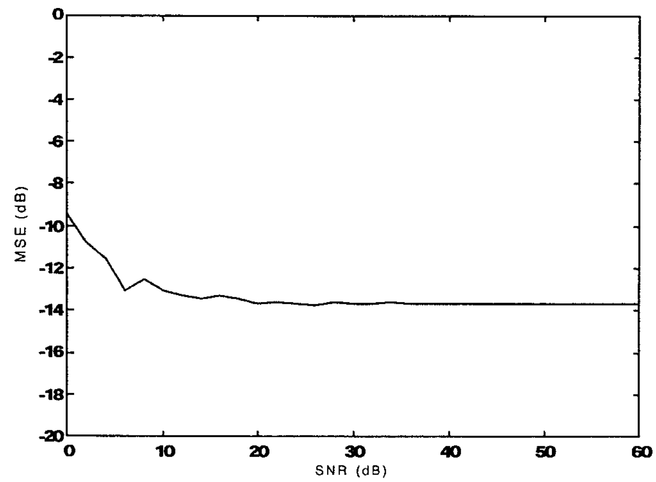

FIG. 7. Noise performance of the CM-SS system with mean-value demodulator.

lation, the time period of the windowed integrator is fixed as $T_s=1/f_s=0.02$ s. Since the variance of the channel noise is controlled, the ratio of the average information signal power P_s to the average noise power P_n (SNR) during the transmission is expressed as SNR$=10\log_{10}(P_s/P_n)$.

Based on the above system parameters, first, we show the transmission of a simple continuous sinusoidal information signal. The information signal given as $s(t)=A\sin(2\pi ft)$ is transmitted directly without any sampling. The amplitude of the information signal $s(t)$ is set at a particular value ($A=1$). The MSE, defined as $D=1/T\int_0^T[s(t)-e(t)]^2dt$, where $e(t)$ is the recovered information signal decoded by the receiver, is used as the performance measure. Figure 6 exhibits the recovery of sinusoidal information signal for SNR$=0$ dB. The performance measure of the present scheme is depicted as in Fig. 7. In this plot, the MSE performance versus different SNR values for sinusoidal information signal transmission is depicted. Apparently, the proposed mean-value demodulation method based communication scheme exhibits good noise performance. It is evident from the figure that even for low SNR values the present scheme performs superiorly well, say, for SNR$=0$ dB, signal recovery with less distortions is achieved. From the simulation results, it is shown that the present chaotic masking communication scheme with the mean-value demodulation (noncoherent) method works well without requiring any additional code recovering schemes, which has potential practical advantages. This method has been further tested both numerically and experimentally using suitable hardware realizations with the logistic map and Chua's circuit equations to transmit different types of analog information signals. Apparently the proposed mean-value demodulation method based analog spread-spectrum communication system exhibits better performance than that of the (coherent) synchronization of chaos-based signal recovery schemes [32].

IV. APPLICATION OF ERGODIC THEORY OF CHAOS FOR DIGITAL SPREAD-SPECTRUM COMMUNICATIONS

The chaotic SS communication system considered here is based on the CPM and the ergodic theory of the chaos-based mean-value estimation method. Assume that s_t is the message signal; the chaotic modulation method uses a chaotic system

$$x_t=f_\theta(x_{t-1},\ldots,x_{t-d}), \tag{6}$$

to modulate s_t by setting $\theta=s_t$ or, in general, $\theta=g(s_t)$, i.e., a function of s_t. Since a one-dimensional system is used in our communication system, $d=1$. We identify suitable θ values for which the system represented by Eq. (6) exhibits chaotic behavior and so the output chaotic signal x_t therefore has a wide bandwidth for spread-spectrum transmission.

For digital communications, the message signal s_t takes on only two values, that is, 0 or 1. Therefore, in the modulation process, only two parameter values are needed to represent the message signal. That is,

$$\theta=\begin{cases}\theta_0 & \text{if} \quad s_t=0,\\ \theta_1 & \text{if} \quad s_t=1.\end{cases} \tag{7}$$

The demodulation process therefore does not require the estimation of a wide range of parameters but now only two values θ_0 and θ_1. To apply the mean-value estimation to the demodulation process, an obvious necessary condition is that θ_0 and θ_1 should not have the same mean value, that is, $M(\theta_0)\neq M(\theta_1)$. Without loss of generality, we can choose θ_0 and θ_1 such that $M(\theta_1)>M(\theta_0)$. The mean-value estimator demodulates the received noisy signal $y_t=x_t+n_t$ by determining whether the parameter used to generate x_r is θ_0 or θ_1. The basic idea of the proposed ergodic CPM (ECPM) communication system is depicted in Fig. 8.

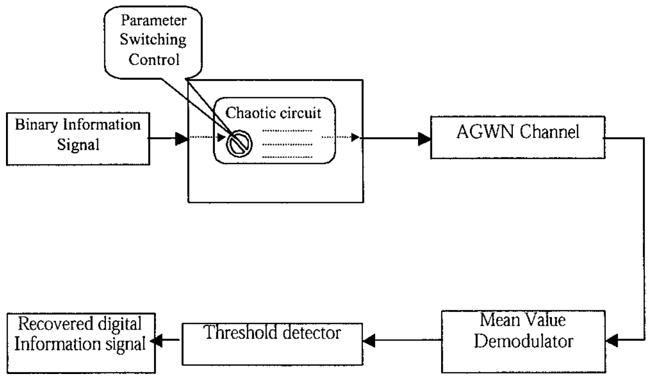

FIG. 8. Block diagram of ergodic chaotic parameter modulation (ECPM) communication system.

Since the Chebyshev map is demonstrated to have a good performance for this ergodic approach in the preceding section, it is employed in the present ECPM communication system. The binary information data s_t (0 or 1) with frequency f_b is spread and modulated by the chaotic signal x_t, and $T_b=1/f_b$ is then the bit duration. The chaotic signal x_t is generated by the Chebyshev map in Eq. (3) with only two possible parameter values: θ_0 or θ_1, that is,

$$x_t=C_{\theta_i}(x_{t-1}), \quad i=0,1, \tag{8}$$

where $t\in[0,T_b]$. If the step size between two adjacent states of the Chebyshev maps is T_x, then the ratio of the chaos generation rate f_x, $f_x=1/T_x$, to the data rate f_b can be considered as the processing gain G of this chaotic SS system. That is,

$$G=\frac{T_b}{T_x}=\frac{f_x}{f_b}. \tag{9}$$

In other words, each data bit is represented by G samples of the chaotic signal generated by the Chebyshev map. The generated chaotic signal is passing through a channel corrupted by AWGN. At the receiver, the noise-corrupted received signal $y_t=x_t+n_t$ is passed to the mean-value estimation based demodulator. Here n_t is the channel noise and is usually assumed to be AWGN. In the proposed demodulator, the mean value of y_t is first estimated using the ensemble average. Since x_t is generated by Eq. (3) and the expectation of an AWGN (n_s) process is zero, the sampled output of the kth bit transmitted signal can then be given as

$$m_k\approx M(\theta_i). \tag{10}$$

To decode the message signal s_t in the kth bit, we need to determine θ_i from m_k. Based on the fact that $M(\theta)$ is monotonic for the Chebyshev map, θ_i can be estimated by $\theta_i \approx M^{-1}(m_k)$. But since θ_i only switches between two values for binary digital communications, the demodulation process can in fact be further simplified to a binary decision process. That is,

$$\hat{s}_t=\begin{cases}1 & \text{if } m_k \text{ is closer to } M(\theta_1),\\ 0 & \text{if } m_k \text{ is closer to } M(\theta_0).\end{cases} \tag{11}$$

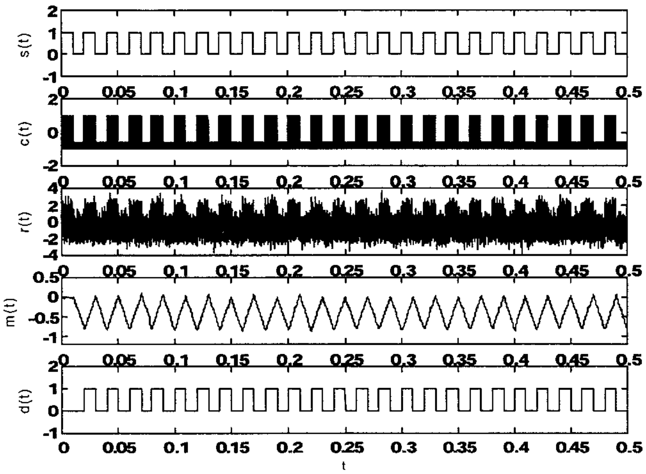

FIG. 9. Signal wave forms in various stages of the ECPM communication system. Here $s(t)$ is the digital information signal, $c(t)$ is the chaos spreading signal, $r(t)$ is the noise-corrupted received signal, $m(t)$ is the mean-value estimated signal, and $d(t)$ is the recovered digital information signal. Additive channel noise is considered for $E_b/N_0=2$ dB.

More precisely, a threshold on the mean value μ_M is set for decision making. That is,

$$\hat{s}_t=\begin{cases}1 & \text{if } m_k>\mu_M,\\ 0 & \text{if } m_k\leqslant\mu_M.\end{cases} \tag{12}$$

In this study, we choose the threshold μ_M to be the midpoint between $M(\theta_0)$ and $M(\theta_1)$, that is, $\mu_M=[M(\theta_1)+M(\theta_0)]/2$. As the asymptotic variance of the estimated mean-value function is equal to $(\delta_\alpha^2+\delta_x^2)/N$, where δ_α^2 is the average power of the chaotic signal, δ_x^2 is the average power of the AWGN and N is the number of samples which is equal to the processing gain in this SS application, i.e., $N=G$. Since G cannot really approach infinity in practice, unless we can estimate the noise mean over the bit duration and subtract it from the signal, it is favorable to maximize the distance between $M(\theta_0)$ and $M(\theta_1)$ to minimize the noise effect. Figure 9 shows the numerical simulation results. Figure 9(a) depicts the binary message signal $s(t)$. Here when $s(t)=0$ (off state), then the chaos system parameter $\theta=(\theta_0)=1.3$. If $s(t)=1$ (on state) then $\theta=(\theta_1)=2.0$ in Eq. (3). Figure 9(b) shows the chaos spreading signal $c(t)$ generated by the map (3) according to the parameter values (θ_0) and (θ_1). Figure 9(c) shows the actual received noise-corrupted signal $r(t)$ at the receiver end. Figure 9(d) depicts the mean-value estimated signal $m(t)$ from the signal $r(t)$. From the signal $m(t)$ after suitable thresholding one can easily decode or recover the digital message signal $d(t)$. But to claim whether the ECPM scheme is useful for communications, its noise performance based on the bit error rate (BER) must be evaluated. To have a better understanding of the effectiveness of the proposed new scheme, we also considered some popular chaotic modulation communication methods, namely, the chaos-shift-keying modulation [8], frequency modulation differential chaos-shift-keying (FMDCSK) scheme [8,22] and the conventional CPM [8,33]

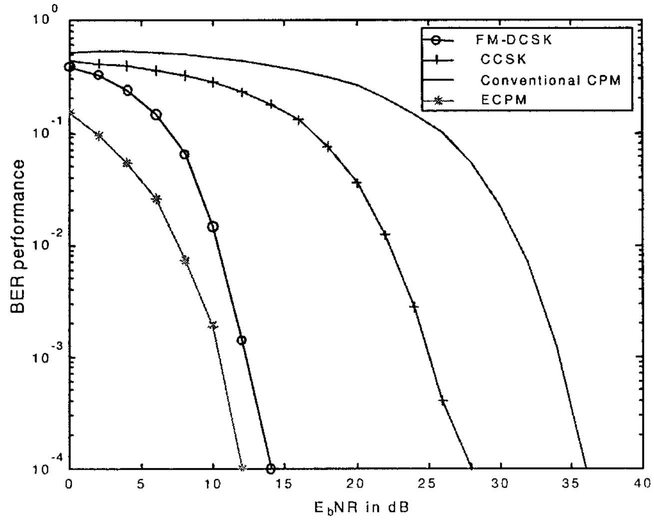

FIG. 10. Comparison of the BER performances of various chaos-based systems in AWGN channel with the noise mean is not exactly zero.

for comparison. Not only are they the most representative chaotic modulation methods, but the FMDCSK has also been shown to be the most effective modulation technique for chaotic communications in the literature so far [23].

Let E_b and N_0 denote the energy per bit and the power spectral density of the AWGN, respectively. We compute the BER versus the E_bNR to evaluate the performance of these chaos modulation communication schemes where E_bNR is the ratio of E_b to N_0. For all these systems, the Chebyshev map is employed as the chaos generator. The FMDCSK uses the frequency modulation for rf modulation, and the others employ the simple amplitude modulation (not shown explicitly in Fig. 8). The frequency is set as 100 kHz and the data rate is 10 Hz. The processing gain is set as $G=1000$, which means 1000 iterations of the chaos signal are generated within each bit duration to represent one binary bit. Because of the short sequence length, the bias in the noise mean will introduce some error in the demodulation process of the ECPM scheme. The BER performances of these four chaos communication systems in AWGN are shown in Fig. 10. Apparently, the proposed ECPM communication system still has the best performance over the compared chaotic modulation schemes. This simple noncoherent communication system is shown to have improved performance than other chaos-based modulation methods. It should be pointed out that there are many advantages of the proposed ECPM communication system. First, it is computationally very simple, and hence the cost for hardware implementation is very low. Second, it does not require the complicated synchronization procedure. As shown in Ref. [23], comparing to other chaos communication schemes, parameter modulation is the only one that is totally unaffected by synchronization errors. Third, the proposed system uses a noncoherent demodulator, which makes the implementation even simpler.

V. CONCLUSIONS

In this paper, we report a method based on the ergodic property of chaotic signals for estimating the bifurcating parameter of a chaotic signal in noise. The proposed method is shown to work effectively even in a very low SNR environment. It is also shown that the proposed method significantly outperforms other conventional techniques in terms of estimation accuracy. In addition, the proposed method is very simple in terms of computational complexity. Those chaotic systems have a monotonic mean-value function over a certain control parameter range of consideration, and can take advantage of this efficient estimation technique. Further an analog SS communication scheme using the chaos additive masking scheme is considered. By exploiting the ergodic property of the chaotic signal it has been shown that using the present noncoherent-type demodulator, efficient signal recovery is possible even for a larger amount of channel noise. Also, based on this ergodic theory of the chaos-based mean-value estimation method, a SS communication scheme called the ergodic chaotic parameter modulation (ECPM) is developed for digital signal transmission. Not only does this ECPM communication system not require any synchronization procedure, but it also has an extremely simple structure for implementation. Although ECPM is a noncoherent scheme, it is shown that it is superior to all other conventional chaotic communication methods in terms of bit error rate performance. In fact, if the signal sequence is sufficient long or the noise mean can be estimated accurately, the BER performance of the ECPM SS system is found to be extremely good. These results strongly support the potential usage of the ergodic theory of chaos-based communication systems for SS applications.

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